

THERMAL AND FORCE LOADS ON THE VEHICLE SURFACE IN HIGH-VELOCITY MOTION IN THE EARTH'S ATMOSPHERE

G. A. Tarnavskii

UDC 518.5:533.6

Consideration has been given to a number of aspects of mathematical modeling of a high-velocity flight in the earth's atmosphere in a wide range of variation of the determining parameters. Super- and hypersonic gas flow past flying vehicles has been investigated based on computer-aided calculations with allowance for its actual properties. Data on the distribution of gasdynamic parameters in the flow field, including thermal and force loads on the surface, have been obtained and analyzed. The issues of applying today's information technologies to archiving scientific knowledge obtained in electronic databases of a specialized Internet center and their dissemination via the Global Network have been discussed.

Introduction. Continuous and rapid progress in the development of computers engineering and the appearance of multiprocessor systems with exclusively high technical data make it possible to substantially extend the mathematical modeling of complex theoretical and applied problems of physical gas dynamics. Today's advances in the creation of high-speed flying vehicles (aircraft and spacecraft) of a new generation and the beginning of their flights in the upper troposphere going into circumferential orbits with altitudes of about 100 km make it necessary to use new physicomathematical models and new algorithms, including parallel counting. Whereas before one has studied sub- and supersonic gas flows under the assumption (model) of the invariability of the physical properties of the gas medium in all the flow zones investigated, modeling in the region of hypersonic velocities requires that this model be extended and the actual properties of the gas, in particular the earth's atmosphere, be allowed for with their change due to different physicochemical processes [1, 2].

However a complete formulation of the problem leads, even now, to extremely high requirements on computers, which exceed today's capabilities. Therefore, one can create universal algorithms and computer programs only when the complete program is decomposed into a number of subproblems: "aerodynamics," "physical processes," and "chemical reactions" (see [3, 4] for details). In addition to improving the efficiency of realization of such computational algorithms on single-processor computers, the structurization of the problem enables one to "parallel" it and to ensure the application of multiprocessor systems to the solution of various gas-dynamic problems (see [4, 5]).

Aim of the Work. The present work is oriented to two aspects of scientific investigations. The first of them is mathematical modeling of high-velocity gas flows: creation of efficient methods and models (reflecting, to a certain degree, the actual processes), construction of computational algorithms, and development of computer programs and obtaining and analyzing, based on them, a new knowledge. The other no less important aim of the work is to apply today's information technologies to support of basic research, to save results in electronic databases, and to disseminate them via specialized scientific centers of the Global Network.

Physical Model. To allow for the physical processes in the gas medium one uses the model of the "effective adiabatic exponent" [6–9]. This method has successfully been used for modeling of a wide range of aerodynamic problems [10–14]. It is based on three basic propositions. First, the problem's "physics" is algorithmically separated from "aeromechanics." Second, these segments (subproblems) in a software system are related by a special quantity — the effective adiabatic exponent γ_{eff} which is a function of the pressure P and temperature T , i.e., a quantity variable throughout the flow region (in a program sense, a 2D or 3D array depending on the dimensions of the problem). It is precisely this fact that fundamentally differentiates γ_{eff} from the "regular" adiabatic exponent γ , a quantity constant throughout the flow region (scalar in a program sense) and with time. Third, γ_{eff} is calculated, in one manner or another, in accordance with the physics of the process (e.g., [15]) or is taken from any tables [16] or electronic databases [17].

Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch of the Russian Academy of Sciences, 6 Prospekt Lavrent'eva, Novosibirsk, 630090, Russia; email: Gennady@Tarnavsky.ru. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 81, No. 2, pp. 338–344, March–April, 2008. Original article submitted July 19, 2006.

TABLE 1. Mach Numbers M of Motion of a Flying Vehicle as Functions of the Altitude H and Velocity V of Flight

V , km/sec	H , km					
	0	20	40	60	80	100
1	2.939	3.389	3.153	3.174	3.539	3.521
2	5.877	6.778	6.305	6.348	7.079	7.054
3	8.816	10.17	9.458	9.521	10.62	10.58
4	11.75	13.56	12.61	12.70	14.16	14.11

Computational Algorithm. In describing gasdynamics, one uses a nonstationary system of differential Navier–Stokes equations that allows for the compressibility, viscosity, and thermal conductivity of the gas medium with the temperature dependence of the corresponding coefficients. The system of equations is closed by the equation of state (generally nonideal). The economical method of splitting by physical processes and space variables is used for numerical integration of this system [3, 18, 19]. The distinctive features of modeling of high-velocity (hypersonic) flow past flying vehicles of different configurations with a possible nonuniqueness (bifurcations) of numerical solutions have been investigated in [10–14], whereas certain and interesting problems of hypersonic flows have been studied and analyzed in [20–22].

Field of Investigations. Computer modeling was carried out in a wide range of variation of the parameters determining the thermogasdynamic structure of flow about a flying vehicle (flow past the bowpart and the lateral surface and flows in the near and far wake). The main factors are flight altitude H and velocity V and the thermal condition on the surface of a body. The quantity H was varied in the range from 0 to 100 km, and V was varied from 1 to 8 km/sec. It is noteworthy that the continuum model holds true nearly throughout this 2D range of parameters (the Knudsen number is quite small), and we can speak of the transient (to a nearly free molecular) regime only when $H > 75$ km. The continuum model is applicable with a sufficient degree of accuracy to correction of the boundary condition in the computational algorithm: switching of the condition of sticking of the flow to the surface to the condition of flow slip and temperature jump. This problem has been investigated in [3] in detail.

In the algorithm, we used two thermal regimes (models): a heat-insulated or isothermal surface of the body. In the latter case, the surface temperature of the body T_w normalized to the stagnation temperature T_0 varied from 0.1 to 1. Furthermore, the configuration of the body's surface is a variable parameter. In this work, we present calculations of flow past two shapes: a drop-shaped DR-6 recoverable orbital capsule and an HB-2 geophysical rocket.

Certain Investigation Results. Mathematical modeling of the process of high-velocity motion of flying vehicles in the atmosphere results in tabular and graphical data on the distribution of the thermogasdynamic parameters in the flow field and on the surface of a flying vehicle. A very wide range of variation of the determining parameters, which is of both theoretical and applied interest, with a large volume of data (even for one variant) necessary for analysis makes the presentation, in review form, of the material obtained optimum and makes it possible, if necessary, to access information of interest and to obtain detailed data on a specific variant of computer-aided calculation.

Two variants of the pattern of flow past a DR-6 recoverable orbital spacecraft and an HB-2 geophysical rockets are presented in Fig. 1 where the fields of the local Mach number in the perturbed flow region in the vicinity of the flying vehicle are shown. In a journal version, only a qualitative analysis of the structure of the perturbed flow region, all gasdynamic parameters, and aerodynamic characteristics of the vehicles is possible, whereas when the specialized electronic databases of the "Aéromekhanika" Internet center are accessed via the Internet (<http://www.SciShop.ru>), a quantitative (in color) analysis is possible.

Table 1 gives the values of Mach numbers M of flight for prescribed H and V values for the actual (standard) earth atmosphere. For the same velocity of flight, the M values are different, with a difference of 20–25%, because of the different values of the velocity of sound at different altitudes. Just as in the case of supersonic velocities, for hypersonic flows, M values can serve as a certain reference point of realizability of a certain flow scenario, although the aeromechanics of flow is substantially related now to the physical processes in the gas medium; these processes are interrelated not only with M but also with specific values of temperatures T and pressures P (and according of density).

Table 2 gives the change in the values of Reynolds numbers Re_∞ (calculated from the parameters of an unperturbed flow for a characteristic dimension of 1 m) as a function of the flight altitude and velocity. The values of

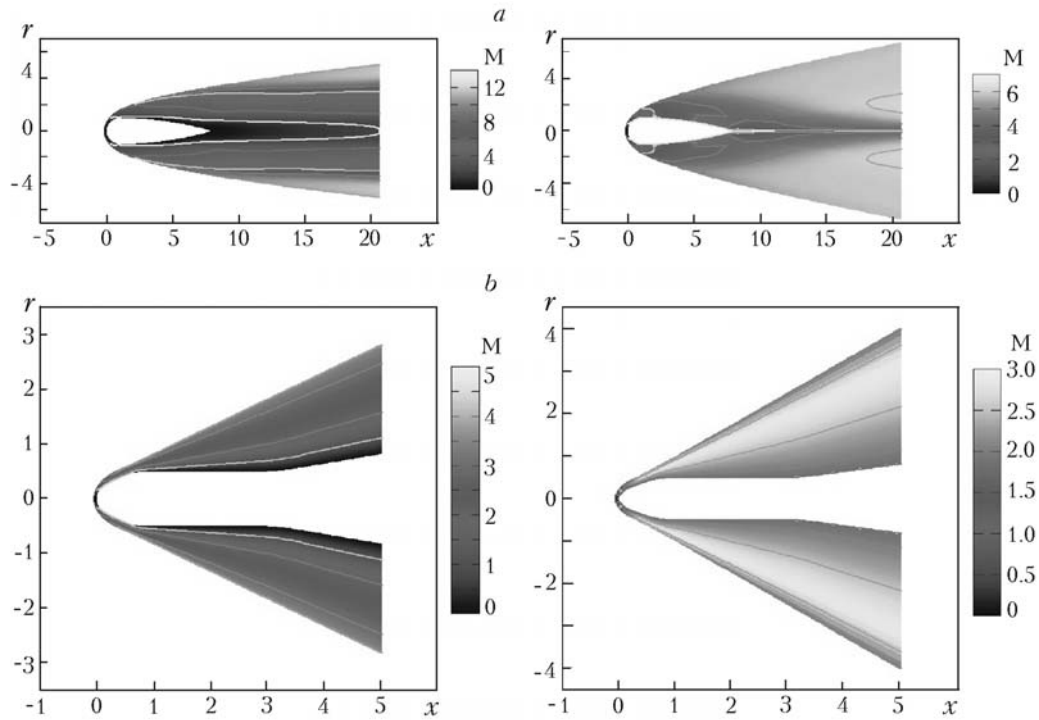


Fig. 1. Perturbed flow region (fields of values of the local Mach number) in the case of flow past the DR-6 recoverable orbital spacecraft (a) and the NB-2 geophysical rocket (b): $H = 70$ km, $V = 6$ km/sec, and $M_\infty = 18.2$ (on the left) and $H = 5$ km, $V = 2$ km/sec, and $M = 6.2$ (on the right).

TABLE 2. Reynolds Numbers Re_∞ of Motion of a Flying Vehicle as Functions of the Altitude H and Velocity V of Flight

V , km/sec	H , km					
	0	20	40	60	80	100
1	$6.846 \cdot 10^7$	$6.255 \cdot 10^6$	$2.496 \cdot 10^5$	$1.956 \cdot 10^4$	$1.398 \cdot 10^3$	$4.241 \cdot 10^1$
2	$1.369 \cdot 10^8$	$1.25 \cdot 10^7$	$4.993 \cdot 10^5$	$3.912 \cdot 10^4$	$2.796 \cdot 10^3$	$8.483 \cdot 10^1$
3	$2.054 \cdot 10^8$	$1.876 \cdot 10^7$	$7.489 \cdot 10^5$	$5.868 \cdot 10^4$	$4.194 \cdot 10^3$	$1.272 \cdot 10^1$
4	$2.738 \cdot 10^8$	$2.502 \cdot 10^7$	$9.986 \cdot 10^5$	$7.824 \cdot 10^4$	$5.593 \cdot 10^3$	$1.697 \cdot 10^1$

Re_∞ are a good reference point for prediction of the regime of flow past a vehicle: a laminar or turbulence, separation or nonseparation regime. Needless to say, the flow regime is also substantially determined by the general shape and type of surface of the vehicle, the level of actual perturbations in the atmosphere, and some other factors. Furthermore, Re_∞ values determine the validity of the employment of the continuum model. The value of Knudsen number Kn is more customary for this purpose; however, Kn and Re_∞ values can be mutually recalculated (see [1, 3, 6]). According to the computational experiments in [3], the limit of applicability of the continuum model is $Re_\infty \approx 50$. The region of the transient regime is located in the Re_∞ range from 50 to 10. We note that a decrease in Re_∞ is determined by the drop in the density of the gas medium rather than by the decrease in the viscosity of the gas. When $Re_\infty < 10$ the continuum model is inadequate to the actual physical processes and one must use other physicomathematical models of a rarefied gas.

One major purpose of mathematical modeling is to obtain the maximum possible exact values of thermogasdynamical parameters in the perturbed region of flow past a flying vehicle. For the general "strategic" review, Tables 3 and 4 give the basic characteristics of flow — temperature T_{w0} and pressure P_{w0} at the nose point of a vehicle (at the stagnation point of the flow) — that are of great practical interest for designing vehicles. It is precisely here that the

TABLE 3. Temperature at the Stagnation Point T (K) on the Surface of the Bowpart of a Flying Vehicle as a Function of the Altitude H and Velocity V of Flight

V , km/sec	H , km					
	0	20	40	60	80	100
0	288	217	250	247	199	197
1	786	714	748	744	696	686
2	2279	2207	2241	2238	2189	2153
3	4767	4695	4729	4726	4677	4599
4	8013	8179	8213	8209	8161	8024

TABLE 4. Pressure at the Stagnation Point P (atm) on the Surface of the Bowpart of a Flying Vehicle as a Function of the Altitude H and Velocity V of Flight

V , km/sec	H , km					
	0	20	40	60	80	100
0	1	$0.5458 \cdot 10^{-1}$	$0.2835 \cdot 10^{-2}$	$0.2168 \cdot 10^{-3}$	$0.1039 \cdot 10^{-4}$	$0.3146 \cdot 10^{-6}$
1	11.83	0.8521	$0.3801 \cdot 10^{-1}$	$0.2918 \cdot 10^{-2}$	$0.1656 \cdot 10^{-3}$	$0.4053 \cdot 10^{-5}$
2	40.18	3.302	0.1477	$0.1139 \cdot 10^{-1}$	$0.6562 \cdot 10^{-3}$	$0.1585 \cdot 10^{-4}$
3	102.1	7.387	0.3308	$0.2548 \cdot 10^{-1}$	$0.1470 \cdot 10^{-2}$	$0.3571 \cdot 10^{-4}$
4	181.3	13.09	0.5861	$0.4498 \cdot 10^{-1}$	$0.2433 \cdot 10^{-2}$	$0.4124 \cdot 10^{-4}$

thermal and force loads are maximum, and knowledge of exact numerical (quantitative) values is necessary for calculating the thermal-protection parameters and strength characteristics of structures (in a number of flow regimes, points T_{\max} and P_{\max} either shift downstream over the surface or "rise" upstream along the line of stagnation of the flow). The T_{w0} and P_{w0} values are actually identical for any dullings of the bowparts.

The first line of each table gives data for an unperturbed earth atmosphere at the corresponding altitude. It is clear that the T_{w0} and P_{w0} maxima are attained for the maximum flight velocity (of those investigated). Temperatures with a value of about 8000 K in the atmosphere lead to a complete dissociation of oxygen and nitrogen molecules and a partial ionization of atoms to form a plasma sublayer near the bowpart of the vehicle. When $V = 2$ km/sec and temperatures of 2000 K are attained, we have a substantial dissociation of just oxygen molecules and partially nitrogen molecules; ionization is slight, particularly at small altitudes. Finally, in the region of regular supersonic velocities $V = 1$ km/sec ($M \sim 3$), neither thermal nor force loads are excessive.

It is noteworthy that for hypersonic velocities and large altitudes of flight, a specific flow structure — a closed viscous shock layer — is formed instead of the comparatively "fine" structures in supersonic flow: the bow shock wave and boundary (thermal and viscous) layers. It is natural that the basic processes determining the flow pattern start and develop precisely in the closed bow viscous shock layer. We should note another flow structure formed in hypersonic flow in the peripheral subrange of determining parameters ($H > 80$ km and $V > 3$ km/sec). Reynolds numbers are quite small here, which is a certain analog of a "very viscous" medium with its own special properties of flow. It is worth further note that the smallness of $Re_{\infty} = \rho_{\infty}VL/\mu_{\infty}$ values here is determined by the low value of the density of the gas medium ρ_{∞} rather than the high value of the viscosity coefficient μ_{∞} . However, in the mathematical model (nonstationary system of dimensionless Navier–Stokes equations of a compressible viscous heat-conducting gas), parameter Re_{∞} is one key parameter determining the solution of the system (both a continuous differential system and a discrete finite-difference one), no matter how this value is attained (the influence of density and viscosity on Re_{∞} is opposite). There, a certain enhancement of "quasiviscous" trends in the structure of the pattern of flow past a vehicle should be expected. Thus, in the case of hypersound an insignificant region of recirculation flow near the body's nose occurs in numerical solutions in the peripheral parametric subrange ($H > 80$ km and $V > 3$ km/sec). Thus, whereas in other parametric ranges, the stagnation point (the very nose of a body) is a point of spreading of the flow, in the above subrange of variations of H and V , the spreading point "rises" toward the incident flow and is located at a certain distance from the body's surface. This resembles the flow structure in ejection of a gas out of the hole in the

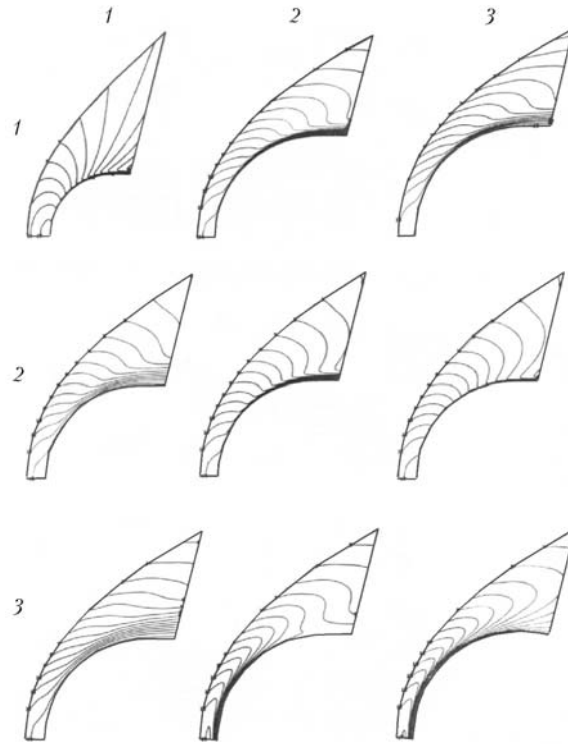


Fig. 2. Change in the topology of temperature isolines in the case of flow past the bowpart of the vehicle (spherical dulling) with variation of determining parameters. The first line: variation $M = 2, 6, \text{ and } 10$ (from left to right), fixed $Re_\infty = 10^4$, and the condition of heat-insulation of the surface. The second line: variation $Re_\infty = 10^3, 10^4, \text{ and } 10^5$ (from left to right), fixed $M = 4$, and the condition of heat-insulation of the surface. The third line: condition of isothermality of the surface with variation $T_w/T_0 = 1.0, 0.5, \text{ and } 0.1$ (from left to right), fixed $M = 6$ and $Re_\infty = 10^4$.

body's nose toward the incident flow ("aerodynamic needle") with the aim of reducing thermal and force loads on the bowpart. However, this issue (possibly associated with passage from the continuous differential problem to a discrete finite-difference one (see also [3, 22]) calls for special study and is beyond the scope of the present work.

A strong shock wave is formed ahead of the vehicle in high-velocity flight. Its intensity ξ (ratio of the pressures behind the front and ahead of it) is substantially dependent on V and to a lesser degree on H . The approximate (somewhat overstated) value of ξ can be obtained from the data of Table 4: $\xi(H, V) = P_{w0}(H, V)/P_{w0}(0, V)$. The absolute values of P are largely determined by the flight altitude. Thus, pressure P_{w0} attains more than 100 atm for $V > 3$ km/sec only at small altitudes; thereupon it sharply drops. The absolute P_{w0} values at large altitudes are very low (compared to 1 atm) even for high velocities, but temperature loads (T_{w0} values) are very high here.

Figure 2 gives, in an integrated manner, the structure of flow past the bowpart of the body and the dependence and change in the flow scenario with variation of the three main parameters: flight numbers M and Re_∞ and the thermal condition on the body's surface. Nine variants of organization of thermal fields, including thermal loads on the bowpart of a vehicle with a spherically shaped dulling, are given here. The temperature distributions (topology of isolines) in the half-plane of symmetry of the problem bounded by the curvilinear surface of the bow shock wave (on top on the left), the spherical dulling of the vehicle, the axis of symmetry of flow, and a certain outlet boundary (on the right) are shown.

Variation of the determining parameters in a wide range (in actual practice, this range is even wider) shows a high diversity of thermal structures. The temperature boundary layer formed along the body's surface and determining heat fluxes into the casing of the flying vehicle can be formed as a layer very narrowly localized near the surface

(variants 1.1, 2.3, and 3.2 in Fig. 2) or occupy a substantial region of peripheral flow (variants 1.3, 2.1, and 3.1). The temperature boundary layer can occur simultaneously along the entire surface (variants 3.2 and 3.3) or can begin to be formed downstream of the stagnation point of the flow (variants 1.1–1.3, 2.1–2.3, and 3.1). Then a low-gradient zone with high absolute values of temperatures exist near the stagnation point (see particularly variants 1.3, 2.1, and 3.1).

The T_{\max} values can be attained in the nose of the body on its surface (most of the shown variants) or can "rise" upstream along the stagnation line (variants 3.2 and 3.3). A very important factor for organizing thermal protection is knowledge of not only the temperature distribution of the gas medium, that is perpendicular to the body's surface, but also longitudinal gradients determining longitudinal thermal loads and accordingly longitudinal stresses on the casing surface (see [23]), particularly in the presence of the maximum heat fluxes (maximum transverse gradients) into the casing against the background of high absolute T_w values. Thus, e.g., the topology of isolines of variant 3.2 demonstrates the presence of a rather vast zone of "thermal rest" in the lower flow subregion in which both the longitudinal and transverse temperature gradients are minimum (virtually zero ones). Such a structure of the thermal field can have an unfavorable effect on the mechanical stresses in the casing due to its high heating in the nose because of the large heat fluxes into the casing in the vicinity of the stagnation point and the minimum heating of the surface downstream with the corresponding occurrence of longitudinal heat fluxes in the casing due to the mechanisms of heat conduction of the material of the vehicle's facing.

Detailed tabular-graphical information on the distributions of all gasdynamic parameters (density, pressure, temperatures, velocity components, and others) in the flow field, including those on the surface of flying vehicles of certain configurations, is available in the "Stream" and "Rocket" databases of the "Aéromekhanika" Internet Center [24–26]. These databases contain several databases (Stream-1, Stream-2, ..., Rocket-1, Rocket-2, ...) each storing 24 record files with the results of computer modeling of any one problem of high-velocity aerodynamics — different classes of streamlined geometries of flying vehicles with variation of determining parameters (velocity and altitude of flight and the temperature regime of the body's surface).

Conclusions. Based on today's computer technologies, we have created an information-computational system of mathematical modeling of high-velocity (super- and hypersonic) flow of a compressible viscous heat-conducting gas past flying vehicles of different configurations. The thermal and force loads on the surface of the dulled bowpart of a body have numerically been investigated in a wide range of variation of determining parameters. A number of the distinctive features of the flow near the stagnation point at large altitudes have been analyzed. A specialized Web resource, the "Aéromekhanika" Center, functioning in the Internet (<http://www.SciShop.ru>), has been created for obtaining and disseminating a new scientific knowledge.

The author expresses his thanks to Professor S. N. Korobeinikov for useful discussions.

This work was carried out with financial support from the Russian Foundation for Basic Research (project Nos. 04-07-90002 and 05-01-00009).

NOTATION

H , flight altitude, km; Kn , Knudsen number; L , characteristic dimension, m; M , Mach number; P , pressure, atm; r , ordinate of the cylindrical coordinate system, reckoned from the problem's axis of symmetry, m; Re , Reynolds number; T , temperature, K; V , flight velocity, km/sec; x , abscissa of the cylindrical coordinate system, reckoned from the body's nose, m; γ , adiabatic exponent; ξ , ratio of the pressures behind the shock and ahead of it; μ , viscosity coefficient; ρ , gas density. Subscripts: eff, effective; max, maximum; w, parameters on the vehicle surface; 0, parameters of stagnation; ∞ , parameters of the unperturbed incident flow.

REFERENCES

1. J. F. Clarke and M. McChesney, *The Dynamics of Real Gases* [Russian translation], Mir, Moscow (1967).
2. a. B. Zel'dovich and Yu. P. Raizer, *Physics of Shock Waves and of High-Temperature Hydrodynamic Phenomena* [in Russian], Nauka, Moscow (1966).
3. V. M. Kovenya, G. A. Tarnavskii, and S. G. Chernyi, *Use of the Method of Splitting in Aerodynamics Problems* [in Russian], Nauka, Novosibirsk (1990).

4. G. A. Tarnavskii and S. I. Shpak, Decomposition of methods and paralleling of the algorithms of solving aerodynamics and physical gas dynamics problems, *Programmirovaniye*, No. 6, 45–57 (2000).
5. G. A. Tarnavskii and V. D. Korneev, Paralleling of the program complex of mathematical simulation of real-gas high-velocity flows, *Avtometriya*, **39**, No. 3, 72–83 (2003).
6. W. G. Hayes and R. F. Probstein, *Hypersonic Flow Theory* [Russian translation], IL, Moscow (1962).
7. R. Feynman, R. B. Leighton, and M. Sands, *Feynman Lectures in Physics*, Vols. 3–4: *Radiation, Waves, Quanta, Kinetics, Heat, Sound* [Russian translation], Mir, Moscow (1976).
8. I. Prigogine and D. Kondepudi, *Modern Thermodynamics. From Heat Engines to Dissipative Structures* [Russian translation], Mir, Moscow (2002).
9. G. A. Tarnavskii and S. I. Shpak, Efficient adiabatic exponent in the problems of hypersonic flow of real gas around bodies, *Teplofiz. Aeromekh.*, **8**, No. 1, 41–58 (2001).
10. G. A. Tarnavskii, Shock waves in gases with different adiabatic exponents before and after the shock, *Vychisl. Metody Programmir.*, **3**, No. 2, 129–143 (2002).
11. G. A. Tarnavskii, Shock-wave structures in real gases: transition between different types of interaction of shocks in the region of the solution nonuniqueness, *Vychisl. Metody Programmir.*, **5**, No. 2, 219–228 (2004).
12. G. A. Tarnavskii, Influence of flow angularities in a hypersonic ramjet diffuser on the formation of the shock-wave structure of the real gas flow, *Inzh.-Fiz. Zh.*, **77**, No. 3, 155–164 (2004).
13. G. A. Tarnavskii, Shock-wave regimes of flow at the inlet to the diffuser of a hypersonic straight-flow air-jet engine: influence of the altitude and speed of flight, *Teplofiz. Vys. Temp.*, **43**, No. 1, 57–70 (2005).
14. G. A. Tarnavskii, Shock-wave flow regimes at entry into the diffuser of a hypersonic ramjet engine: influence of physical properties of the gas medium, *Inzh.-Fiz. Zh.*, **79**, No. 4, 69–80 (2006).
15. G. A. Tarnavskii and S. I. Shpak, Techniques of calculation of the effective adiabatic exponent in computer modeling of hypersonic flows, *Sib. Zh. Industr. Mat.*, **4**, No. 1(17), 177–197 (2001).
16. N. B. Vargaftik, *Handbook on the Thermophysical Properties of Gases and Liquids* [in Russian], Fizmatgiz, Moscow (1963).
17. A. N. Migun, E. A. Matveichik, A. P. Chernukho, and S. A. Zhdanok, Object-oriented approach to simulation of chemical reaction kinetics, *Inzh.-Fiz. Zh.*, **78**, No. 1, 153–158 (2005).
18. G. A. Tarnavskii and S. I. Shpak, Problems of numerical simulation of a supersonic laminar-turbulent flow around finite-size bodies, *Mat. Modelir.*, **10**, No. 6, 53–74 (1998).
19. G. A. Tarnavskii and S. I. Shpak, Calculation of heat fluxes in supersonic viscous gas flow around bluff bodies, *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 3, 179–183 (1999).
20. V. F. Volkov and G. A. Tarnavskii, Disturbance of symmetry and hysteresis of stationary and quasi-stationary solutions of the Euler and Navier–Stokes equations, *Zh. Vych. Mat. Mat. Fiz.*, **41**, No. 11, 1742–1750 (2001).
21. G. A. Tarnavskii and S. I. Shpak, Certain aspects of computer simulation of hypersonic flows: stability, nonuniqueness, and bifurcation of numerical solutions of the Navier–Stokes equations, *Inzh.-Fiz. Zh.*, **74**, No. 3, 125–132 (2001).
22. G. A. Tarnavskii, G. S. Khakimzyanov, and A. G. Tarnavskii, Modeling of hypersonic flows: influence of the starting conditions of the algorithm on the final solution in the vicinity of bifurcation points, *Inzh.-Fiz. Zh.*, **76**, No. 5, 54–60 (2003).
23. S. N. Korobeinikov, *Nonlinear Deformation of Solid Bodies* [in Russian], Izd. SO RAN, Novosibirsk (2000).
24. G. A. Tarnavskii, A. G. Tarnavskii, and K. V. Gilev, Informational-computational Internet-Center "Aeromechanics." The firstline: program complex "Shock," *Vychisl. Metody Programmir.*, **6**, No. 1, 27–48 (2005).
25. G. A. Tarnavskii, A. G. Tarnavskii, A. V. Aliev, and S. B. Zhibinov, Informational-computational complex "Flow" of the Internet-Center for dissemination of scientific knowledge, *Infosfera*, No. 34, 35–38 (2007).
26. G. A. Tarnavskii, A. V. Aliev, and A. G. Tarnavskii, Computer simulation in aeromechanics: program complex "Flow-5," *Aviakosm. Tekh. Tekhnol.*, No. 4, 27–38 (2007).